**Examples on how to calculate combined standard uncertainty**

It is very important for anyone interested in the evaluation of measurement uncertainty to fully understand the very basic principles in calculating the combined standard uncertainty.

When we are handling a set of standard uncertainties from an independent step in an analytical method to be combined to form the standard uncertainty of that step, we can calculate the combined independent uncertainties as the root sum of their squares. Diagrammatically, this looks like Pythagoras’s theorem, as shown below:

**Figure 1:** Simple rule for combining independent uncertainty components

\[ u_{a} = \sqrt{u_{a}^2 + u_{b}^2} \]

The followings are the simple calculation rules that can be applied:

a. If a quantity \( x_i \) is simply added to or subtracted from all the others to obtain the final result \( y \), the contribution to the uncertainty in \( y \) is simply the uncertainty \( u(x_i) \) in \( x_i \). In other words, if variables are added or subtracted, the uncertainties combine as simple standard uncertainties. See the following example.

**Example 1:**

In the determination of using a 10-ml bulb pipette to transfer 10-ml sample volume for analysis, we have three significant uncertainty contributors:

- Manufacturer’s uncertainty (U) of ±0.05ml for the 10-ml pipette: the standard uncertainty \( u_m \) therefore is estimated to be \( 0.05/\sqrt{3} \) assuming a rectangular distribution because no confidence level is given in the product catalog.
- Laboratory repeatability of using this 10-ml pipette with a standard uncertainty $u_r$ as expressed as its standard deviation after say 10 repeated experiments, say 0.060.08ml

- Uncertainty contribution from water volume expansion at the laboratory room temperature of 25°C whilst the pipette has been calibrated by the manufacturer at 20°C. We take the coefficient of water expansion as 0.000214 per °C per ml from a book of constants. Hence, the uncertainty of volume expansion is estimated to be $10\text{ml} \times 0.000214 \times (25-20)°\text{C}$ or 0.0107. The standard uncertainty due to the water volume expansion, $u_v$, is $0.0107/\sqrt{3}$ or 0.06ml by assuming a rectangular distribution.

Therefore, the combined standard uncertainty of this 10-ml pipette is

$$u_p^2 = \sqrt{u_m^2 + u_r^2 + u_v^2} = \sqrt{0.05^2 + 0.06^2 + 0.0107^2} = 0.08$$

Note that this manner of calculation applies only on uncertainty components with the same type of measuring unit, being ml in this example.

b. If a quantity is multiplied or divides, the rest of the expression for $y$, the contribution to the relative uncertainty in $y$, $u(y)/y$, is the relative uncertainty $u(x_i)/x_i$ in $x_i$. In other words, if variables are multiplied or divided, their uncertainties combine as relative uncertainties.

**Example 2:**

In an analysis of Cd pollutant in dried soil sample by atomic absorption spectrophotometric method, the equation of calculating the Cd content in mg/kg, $y$, is

$$y = \frac{C_y \times V \times D}{M}$$

where

- $C_y$ is the concentration of the prepared acid digested solution (after $D$ dilution) read from the calibration graph
- $V$ is the volume of the acid digested sample solution
- $M$ is the mass of dried soil sample taken for acid digestion

Assume we have got the following tabulated analytical data and their respective calculated relative standard uncertainty:
By calculation, the Cd concentration in the dried soil sample analyzed, $y$, was found to be 163 mg/kg. The relative combined standard uncertainty $u_y/y$ is thus calculated as follows:

$$u_y/y = \sqrt{0.061^2 + 0.014^2 + 0.0011^2 + 0.02^2} = 0.066$$

Therefore, the combined standard uncertainty $u_y = 163 \times 0.066 = 10.7$ mg/kg

In conclusion, the following table summarizes the simple rules for uncertainty calculations:

<table>
<thead>
<tr>
<th>Result $y$ calculated from</th>
<th>Uncertainty $u_y$ in $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = a + b$ or $y = a - b$</td>
<td>$u_y = \sqrt{u_a^2 + u_b^2}$</td>
</tr>
<tr>
<td>$y = a \times b$ or $y = a/b$</td>
<td>$\frac{u_y}{y} = \sqrt{\left(\frac{u_a}{a}\right)^2 + \left(\frac{u_b}{b}\right)^2}$</td>
</tr>
</tbody>
</table>