How to use Excel® spreadsheet for the Anderson–Darling (AD) test statistic

AD test statistic – the aim

- AD test statistic can replace the traditional hypothesis testing（e.g. Ho: $\mu 1 = \mu 2$; H1: $\mu 1 \neq \mu 2$）
- It is used to check if the data collected obey a certain probability distribution function (PDF)（e.g. using Student's t test to verify normal distribution）
- AD test statistic has the following hypotheses：
  - Ho： data obey a targeted PDF
  - H1： data do not obey a targeted PDF
- The $A_s^{2^*}$ and $A_{MR}^{2^*}$ values obtained can verify the data are normally distributed and independent, respectively

Equations of AD test statistic

$$A^2 = -\frac{\sum_{i=1}^{n}(2i-1)\left[\ln(p_i)+\ln(1-p_{n+1-i})\right]}{n} - n$$

$$A^{2*} = A^2\left(1+\frac{0.75}{n}+\frac{2.25}{n^2}\right)$$

- A^2 is AD normal distribution estimated statistic （against standard deviation s, and moving range MR）
- A^{2*} is AD adjusted statistic value （ against standard deviation s, and moving range
- p_i is the probability of data i to obey PDF
- n is the total data in question（$n > 15$)

Outliers in the data set must be removed before using the above AD equation for As^{2*} and AMR^{2*}

Let the non-outlier data set is

$$(x_1, x_2, x_3, \ldots x_i \ldots., x_n)$$

- Mean： $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
- Std Deviation： $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$
- Absolute moving range : $|MR_i| = |x_{i+1} - x_i|$
- Intermediate std deviation $s_{R'} = \frac{\overline{MR}}{1.128}$ (equivalent to Std uncertainty of the measurments）
- Use MS Excel® spreadsheet to calculate pi value “=NORMSDIST()”

Interpretations of AD test values

- *a*） $As^{2*} < 1.0$ and $AMR^{2*} < 1.0$ ： To accept Ho, i.e. the data set is in accordance to normal distribution and the data are independent; one may use these data to build a control chart
- *b*） $As^{2*} > 1.0$ and $AMR^{2*} > 1.0$ ： indicating that under 99％ confidence，the data are not normally distribution；reject Ho
- *c*） $As^{2*} < 1.0$ and $AMR^{2*} > 1.0$ ： indicating the data are normally distributed but have certain trend in the data collection. One has to conduct further testing or investigate the reasons behind this observation

An Excel® spreadsheet example

Anderson Darling 统计检验量

$$A^2 = -\frac{\sum_{i=1}^{n}(2i-1)\left[\ln(p_i)+\ln(1-p_{n+1-i})\right]}{n} - n \qquad A^{2*} = A\left(1+\frac{0.75}{n}+\frac{2.25}{n^2}\right)$$

$\omega_i(s) = (x_i - \bar{x})/s$ $\omega_i(MR_i) = (MR_i - \overline{MR})/s(MR)$

$p_i := NORMDIST(\omega_i)$

序	Original Data 原数据			Standardized Data标准化数据				基于标准差, s						基于移动极差标准差, sMR					
	数据	移动极差	升序	升序	降序	升序	降序	升序		降序				升序		降序			
i	I_i	$\lvert MR_i\rvert$	I_i	$\omega_i(s)$	$\omega_i(s)$	$\omega_i(MR_i)$	$\omega_i(MR_i)$	p_i	$\ln(p_i)$	p_{n+1-i}	$\ln(1-p_{n+1-i})$	$2i-1$	A_i	p_i	$\ln(p_i)$	p_{n+1-i}	$\ln(1-p_{n+1-i})$	$2i-1$	A_i
1	24.81	/	20.99	-1.5973	1.9804	-1.3585	1.6843	0.0551	-2.8985	0.9762	-3.7368	1	-6.6354	0.0872	-2.4400	0.9539	-3.0778	1	-5.5178
2	21.15	3.66	21.00	-1.5916	1.4483	-1.3536	1.2317	0.0557	-2.8871	0.9262	-2.6068	3	-16.4817	0.0879	-2.4313	0.8910	-2.2162	3	-13.9424
3	25.98	4.83	21.15	-1.5067	1.4086	-1.2814	1.1980	0.0659	-2.7189	0.9205	-2.5324	5	-26.2565	0.1000	-2.3024	0.8845	-2.1589	5	-22.3064
4	23.53	2.45	22.10	-0.9689	1.3351	-0.8241	1.1355	0.1663	-1.7940	0.9091	-2.3977	7	-29.3416	0.2050	-1.5850	0.8719	-2.0550	7	-25.4798
5	26.30	2.77	22.30	-0.8557	1.2275	-0.7278	1.0440	0.1961	-1.6292	0.8902	-2.2089	9	-34.5433	0.2334	-1.4551	0.8518	-1.9089	9	-30.2756
6	23.23	3.07	22.31	-0.8500	0.9275	-0.7229	0.7888	0.1977	-1.6212	0.8232	-1.7325	11	-36.8912	0.2349	-1.4488	0.7849	-1.5366	11	-32.8392
7	22.31	0.92	22.51	-0.7368	0.5652	-0.6267	0.4807	0.2306	-1.4670	0.7140	-1.2518	13	-35.3450	0.2654	-1.3264	0.6846	-1.1540	13	-32.2447
8	24.55	2.24	22.84	-0.5500	0.4916	-0.4678	0.4181	0.2912	-1.2339	0.6885	-1.1663	15	-36.0035	0.3200	-1.1395	0.6621	-1.0849	15	-33.3662
9	23.18	1.37	23.13	-0.3858	0.4180	-0.3282	0.3555	0.3498	-1.0504	0.6620	-1.0848	17	-36.2977	0.3714	-0.9905	0.6389	-1.0186	17	-34.1542
10	26.37	3.19	23.18	-0.3575	0.2425	-0.3041	0.2063	0.3603	-1.0207	0.5958	-0.9059	19	-36.6046	0.3805	-0.9662	0.5817	-0.8716	19	-34.9173
11	23.96	2.41	23.23	-0.3292	0.0840	-0.2800	0.0714	0.3710	-0.9916	0.5335	-0.7624	21	-36.8345	0.3897	-0.9423	0.5285	-0.7518	21	-35.5757
12	23.13	0.83	23.46	-0.1990	-0.0405	-0.1693	-0.0345	0.4211	-0.8648	0.4838	-0.6613	23	-35.1019	0.4328	-0.8375	0.4863	-0.6660	23	-34.5811
13	27.31	4.18	23.53	-0.1594	-0.1594	-0.1356	-0.1356	0.4367	-0.8286	0.4367	-0.5739	25	-35.0616	0.4461	-0.8073	0.4461	-0.5907	25	-34.9498
14	22.51	4.80	23.74	-0.0405	-0.1990	-0.0345	-0.1693	0.4838	-0.7260	0.4211	-0.5467	27	-34.3620	0.4863	-0.7210	0.4328	-0.5670	27	-34.7775
15	22.30	0.21	23.96	0.0840	-0.3292	0.0714	-0.2800	0.5335	-0.6283	0.3710	-0.4636	29	-31.6666	0.5285	-0.6378	0.3897	-0.4939	29	-32.8168
16	23.74	1.44	24.24	0.2425	-0.3575	0.2063	-0.3041	0.5958	-0.5178	0.3603	-0.4468	31	-29.9045	0.5817	-0.5418	0.3805	-0.4789	31	-31.6413
17	26.17	2.43	24.55	0.4180	-0.3858	0.3555	-0.3282	0.6620	-0.4125	0.3498	-0.4305	33	-27.8169	0.6389	-0.4480	0.3714	-0.4643	33	-30.1050
18	20.99	5.18	24.68	0.4916	-0.5500	0.4181	-0.4678	0.6885	-0.3732	0.2912	-0.3441	35	-25.1079	0.6621	-0.4124	0.3200	-0.3856	35	-27.9307
19	21.00	0.01	24.81	0.5652	-0.7368	0.4807	-0.6267	0.7140	-0.3368	0.2306	-0.2622	37	-22.1632	0.6846	-0.3789	0.2654	-0.3085	37	-25.4325
20	25.45	4.45	25.45	0.9275	-0.8500	0.7888	-0.7229	0.8232	-0.1946	0.1977	-0.2202	39	-16.1780	0.7849	-0.2422	0.2349	-0.2677	39	-19.8863
21	23.46	1.99	25.98	1.2275	-0.8557	1.0440	-0.7278	0.8902	-0.1163	0.1961	-0.2183	41	-13.7182	0.8518	-0.1605	0.2334	-0.2658	41	-17.4751
22	24.24	0.78	26.17	1.3351	-0.9689	1.1355	-0.8241	0.9091	-0.0953	0.1663	-0.1819	43	-11.9200	0.8719	-0.1371	0.2050	-0.2294	43	-15.7564
23	24.68	0.44	26.30	1.4086	-1.5067	1.1980	-1.2814	0.9205	-0.0828	0.0659	-0.0682	45	-6.7962	0.8845	-0.1227	0.1000	-0.1054	45	-10.2627
24	22.84	1.84	26.37	1.4483	-1.5916	1.2317	-1.3536	0.9262	-0.0766	0.0557	-0.0574	47	-6.2973	0.8910	-0.1154	0.0879	-0.0920	47	-9.7510
25	22.10	0.74	27.31	1.9804	-1.5973	1.6843	-1.3585	0.9762	-0.0241	0.0551	-0.0567	49	-3.9591	0.9539	-0.0472	0.0872	-0.0912	49	-6.7791

$\bar{x}_i =$ 23.81 2.343 $= \overline{MR}$

s = 1.767 2.08 = sR' = sMR

Σ Ai(s)= -631.2885
$A^2(s)=$ 0.2515
$A^{2*}(s)=$ 0.2600

Σ Ai(MR)= -632.7646
$A^2(MR)=$ 0.3106
$A^{2*}(MR)=$ 0.3210

An Excel® spreadsheet example will be provided in the next blog.

Happy calculations !