

## Must we always use probability $p = 0.05$ for 95% confidence interval?

When we have carried out a large number of replicated analyses on a sample to obtain a mean  $\bar{X}$  and standard deviation  $s$ , we usually express the 95% confidence interval as:  $X = \bar{X} \pm 1.96 \times s$  under a normal probability distribution with reference to the central limit theorem where the  $z$ -score is 1.96. If we are interested in the variability of sample means from a population, then we shall estimate its standard error  $SE$  which is  $= \sigma / \sqrt{N}$  where  $\sigma$  is the population standard deviation with  $N$  number of samples analyzed where  $N > 30$ . Hence, the 95% confidence interval of the population mean  $\mu$  is given by:

$$\mu = \bar{X} \pm 1.96 \times SE$$

As such, we know that the mean is always in the center of the confidence interval. With 95% of confidence interval to contain the population mean, we can assume this confidence interval contains the *true* mean or the *true* value of the test parameter in the sample analyzed. Therefore, if the interval is small, the sample mean must be very close to the true mean whilst conversely, if the confidence interval is very wide, we can conclude that the sample mean could be very different from the true mean, indicating that it is a bad representation of the population.

For a 99% confidence interval, the  $z$ -score for  $(1-0.99)/2 = 0.005$  when read from the normal distribution table is 2.58. For a 90% confidence interval, the  $z$ -score for  $(1-0.90)/2 = 0.05$ , which from the normal distribution table is 1.64.

The question we would like to ask is: why would we traditionally prefer to use 95% confidence interval instead of other confidence levels?

In my opinion, 95% confidence interval translates to a 5% error. In other words, a given 95% confidence interval of mean would effectively tell us that out of 20 repeated tests, we can expect only 1 test result the very most which is outside this defined range. It is definitely more convincing that saying out of 10 repeated tests, the most only 1 test result is outside the defined range. A 99% confidence interval on the other hand gives too narrow the range which gives rise to a higher risk of wrongly assessing the *true* mean result.