Is an expanded uncertainty synonym to a confidence limit or interval?

Many practicing chemists have been confused by the meanings of "expanded uncertainty" and "confidence limit" or "confidence interval". The reason is that both terms have similar expression of $X \pm Y$ where $Y = U$ for expanded uncertainty and $Y = CL$ for confidence limit. Are they referring to a similar concept?

Expanded uncertainty is derived from a product of a coverage factor k and the combined standard uncertainty after considering the various valid uncertainty budgets in a test method. The budgets can be from the uncertainty components of each analytical steps leading to the final result outcome (GUM approach). The contributions of the standard uncertainties may come from Type A evaluation of uncertainty (own derived statistical data) or Type B evaluation of uncertainty (derived from a given uncertainty U by say the supplier, or from past experiences or scientific judgment with different probability distribution functions considered), or a combination of both types.

The expanded uncertainty can also be obtained from a so-called top-down approach by considering the overall performance of the test method in terms of its reproducibility, repeatability and trueness.

The intention of evaluating measurement uncertainty is to establish a numerical interval of the test result which contains the *true value* of the analyte with a certain level of confidence, say 95% with a coverage fact k = 2. Indeed, expanded uncertainties are computed by assuming a combined uncertainty estimate with infinite degrees of freedom ($\nu = \infty$) for a combined error that is normally distributed.

A confidence interval however gives a range of targeted analyte values of a *population* with a certain confidence level, say 95%. This is because we cannot do a 100% measurement on a population but only on its samples.

The values at each end of the intervals are called the confidence limits CL. This CL is a good estimate of the population mean value μ. Repeated measurements are made on several samples and the standard error of the mean of these samples is then estimated. If the number of repeats (n) is less than 30 (degree of freedom ν being 30-1 or 29), a Student's t distribution with the following expression can be adopted for the confidence limit CL:

$$CL = \bar{x} \pm t\frac{s}{\sqrt{n}}$$

where

s is the standard deviation of the sample mean $\bar{x}$ after *n* repeats, and

$\frac{s}{\sqrt{n}}$ is called the standard error of the mean.

Note : this expression does not refer to repeated measurement of the same sample, but repeated random sampling with measurements done on each sample.

And, if the number of samples (n) is sufficiently large, the conditions of the Central Limit Theorem are satisfied and the following equation can then be applied for the CL under a normal distribution:

$$CL = \mu \pm z\frac{\sigma}{\sqrt{n}}$$

where

μ is the population mean

z is the z-score from a normal distribution table

σ is the mean standard deviation after analysis of a large number of samples, n.

From the above discussion, it is obvious that expanded uncertainty of a test procedure does not equate to the confidence limit or interval found in the method for a target analyte. CL attempts to claim that "*were this test method to be repeated on multiple samples, the estimated confidence limit would encompass the true population mean value, say 95% of the time.*" In other words, if 20 samples were analyzed, there would be one chance that the test result is found outside the confidence interval estimated. Measurement uncertainty on the other hand provides a range of values covering the true value of the analyte in the test method with 95% confidence (k=2) by evaluating the various uncertainty components involved.