Confidence Intervals:

How many measurements should you make?

When establishing confidence interval from experimental data, we are sometimes told to make seven measurements? Why seven?

The magical nature of this number actually stems from the fact that the standard deviation of seven results is just greater than the 95% confidence interval of the mean value. How is it so?

Let's first take a look at Student's t-equation which is

$$\mu = \bar{x} \pm t_{\alpha,v=n-1} \frac{s}{\sqrt{n}} \qquad [1]$$

When n = 7 measurements, we have v = (7–1) or 6 degrees of freedom, √7 = 2.65, and at α = 0.05, the $t_{\alpha,v}$ = 2.45, thus the confidence interval for 7 measurements is (2.45)s/(2.65) or 0.925s. In here, we see the constant factor is 0.925, leaving s as about the 95% confidence of the mean.

In fact, the answer to how many measurements you should make is answered by how precise you want your result to be.

If the population standard deviation (σ) is known, then if we want to make sure that the error in our mean, (i.e. the difference between the experimental mean and the population mean that would be obtained by an infinite number of repeats) is no more than error ε with a probability of α, then we can derive the error from the Central Limit Theorem equation

$$\varepsilon = \mu - \bar{x} = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \qquad [2]$$

When equation [2] is rearranged, we get the number of measurements, n as:

$$n = \left(\frac{z_{\alpha/2}\sigma}{\varepsilon}\right)^2 \qquad [3]$$

In fact, Equation [3] is quite instructive. It tells us that we must do more experiments if we want a smaller uncertainty (ε), a higher probability (α), or have a smaller standard deviation of our experiments.

What will happen when we do not know the population statistics μ and σ, but only have data from some preliminary experiments? Are we in something of a dilemma?

No. Firstly, we may already have some experimental data, and secondly, let's take a look at equation [1] again, where we see that the equivalent expression to equation [3] is

$$n = \left(\frac{t_{\alpha,v=n-1}s}{\varepsilon}\right)^2 \qquad [5]$$

where s is the sample standard deviation.

The challenge with using equation [5] is that we need to know n in order to calculate the degrees of freedom, to give the Student's t–value.

Actually what we can do is to make an initial guess at n to give t which is then put back in n, and so on.

After the experiments are performed, it may be necessary to recalculate n to take into account the new value for the standard deviation. In practice, we are usually interested in ball-park figures, for example, 5% with anywhere between 4 and 6% being acceptable. Therefore the process is not really that tedious after all.