Control chart methods – Part II

There are important properties of a quality control chart in test laboratories. The first is the speed in which it detects that a change in the analyte mean in the analysis process has occurred. Its laboratory control sample's (LCS) moving range, R, which is $R = x_{i+1} - x_i$ over a period of time can give us its average $\bar{R}$. We can then find its intermediate reproducibility standard uncertainty, $u = \bar{R}/1.128$. This is a top down approach in the analytical uncertainty evaluation that is easy and fast, provided the laboratory practices a systematic quality control protocol in its routine analytical work.

TH Shewhart control chart mentioned in the previous article depends on the value of $\frac{\sigma}{\sqrt{n}}$ and relies on quite a number of measurements over time (known as average run length ARL) to detect any particular change in the analyte mean. In fact, it can be shown that for a significant change equal to $1 \times \frac{\sigma}{\sqrt{n}}$ to occur, the ARL is about 50 when only the criterion of action line based on control limits is used. In other words, about 50 measurements are to be there before a value falls outside the action limit lines and the process will be stopped for investigation.

This ARL of 50 measurements sounds long but it can be reduced significantly by using a different type of control chart, a CUSUM (cumulative sum) chart. The general equation used in CUSUM may be difficult to apprehend but the following example gives a clearer way how the calculation is done in this approach.

Table 1: An example of experimental data for CUSUM calculation

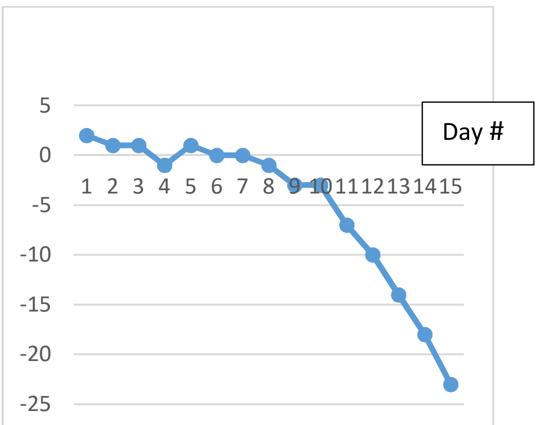
Target Value μ = 80 Std Deviation σ = 2.5

Day #	Sample mean	Sample mean - target value	CUSUM
1	82	2	2
2	79	-1	1
3	80	0	1
4	78	-2	-1
5	82	2	1
6	79	-1	0
7	80	0	0
8	79	-1	-1
9	78	-2	-3
10	80	0	-3
11	76	-4	-7
12	77	-3	-10
13	76	-4	-14
14	76	-4	-18
15	75	-5	-23

Table 1 above shows a set of data for CUSUM calculation. The last two columns of the table show the sum of the deviation of the sample means from the target value is carried forward cumulatively. For example, in day #1, the sample mean – target value = 82 – 80 = 2 whilst the CUSUM = 2 – 0 = 2; in day #2, the sample mean – target value = 79 – 80 = -1 whilst the CUSUM = 2 + (-1) = 1, *etc*. Care must be paid to the signs of the deviations.

If the analytical process is under good control, we expect to have positive and negative deviations from the target value to be equally likely and the CUSUM should oscillate about zero. If the analyzed target value changes, the CUSUM will move away from zero. This is obvious when we plot the CUSUM chart for the Table 1 data:

Figure 1: CUSUM chart for Table 1 data



The above CUSUM chart shows that the analytical mean seems to fall after the seventh day, so the CUSUM becomes more and more negative, indicating the process is going to be out of control, but its Shewhart control chart still shows the data collected are still within its lower warning limit. (see Figure 2).

Figure 2: Shewhart chart for Table 1 data