Practical way to calculate combined uncertainty by MS Excel spreadsheet

Using an ordinary spreadsheet to calculating a combined uncertainty saves a great deal of time and is particularly useful for checking and updating with new data whenever available.

The Eurachem/CITAC Guide (3rd edition) *Quantifying Uncertainty in Analytical Measurement* has given a number of spreadsheet calculation examples. The approach using the MS Excel spreadsheet is summarized as described below.

Assume that the test result y is a function of three input parameters p, q and r, i.e. $y = f(p, q, r)$, such as: $y=(p \times q)/r$.

Let's set up a basic spreadsheet as in Figure 1:

In Figure 1, enter the values of p, q, r and the formula for calculating y in cell A6 of the spreadsheet and the respective values of standard uncertainties $u(p)$, $u(q)$ and $u(r)$ in column B. Copy cell A6 across to C6:E6.

Figure 1: Entries of raw data

	A	B	C	D	E
1		u			
2	p	$u(p)$	p	p	p
3	q	$u(q)$	q	q	q
4	r	$u(r)$	r	r	r
5					
6	$y=f(p,q,r)$		$y=f(p,q,r)$	$y=f(p,q,r)$	$y=f(p,q,r)$
7					
8					

Add $u(p)$ to p in cell C2, $u(q)$ to q in cell D3, and $u(r)$ to r in cell E4 as in Figure 2 below. Upon recalculating the spreadsheet automatically by the software, cell C6 then becomes $f(p+u(p),q, r)$ [denoted by $f(p',q,r)$], and so are the others.

Figure 2: Entries of individual standard uncertainty contributions

	A	B	C	D	E
1		u	p	q	r
2	p	$u(p)$	$p+u(p)$	p	p
3	q	$u(q)$	q	$q+u(q)$	q
4	r	$u(r)$	r	r	$r+u(r)$
5					
6	$y=f(p,q,r)$		$y=f(p',q,r)$	$y=f(p,q',r)$	$y=f(p,q,r')$
7					
8		Diff :	=C6-A6	=D6-A6	=E6-A6
9		Squared Diff:	=C8^2	=D8^2	=E8^2

The row 8 of Figure 2 calculates the difference between the calculated y values in columns C, D and E individually and the originally calculated y value in A6. The row 9 shows the squared differences of columns C, D and E.

To obtain the combined standard uncertainty in y, i.e. $u(y)$, just sum up all these individual squared differences in row 9 and take a square root of the sum, as shown in Figure 3.

Figure 3: Calculation of combined standard uncertainty

	A	B	C	D	E
1		*u*	*p*	*q*	*r*
2	*p*	*u(p)*	*p+u(p)*	*p*	*p*
3	*q*	*u(q)*	*q*	*q+u(q)*	*q*
4	*r*	*u(r)*	*r*	*r*	*r+u(r)*
5					
6	*y=f(p,q,r)*	*u(y)* *	*y=f(p',q,r)*	*y=f(p,q',r)*	*y=f(p,q,r')*
7					
8		Diff :	=C6-A6	=D6-A6	=E6-A6
9		Squared Diff:	=C8^2	=D8^2	=E8^2
10	* The formula is =SQRT(SUM(C9:E9)				
11					

In fact, the contents of the cells C8, D8 and E8 show the contributions $u_i(y)=c_i u(x_i)$ of the individual uncertainty components to the uncertainty in y, making it easy to see which components are significant. It may be noted that c_i is called the sensitivity coefficients of standard uncertainty $u(x_i)$.

A screen picture captured from the Eurachem/CITAC Guide on an example of the calculation table is shown below.

Quantifying Uncertainty **Example A4**

Table A4.5: Uncertainties in pesticide analysis

	A	B	C	D	E
1			Precision	Bias	Homogeneity
2		value	1.0	0.9	1.0
3		uncertainty	0.27	0.043	0.2
4					
5	Precision	1.0	1.27	1.0	1.0
6	Bias	0.9	0.9	0.943	0.9
7	Homogeneity	1.0	1.0	1.0	1.2
8					
9	P_{op}	1.1111	1.4111	1.0604	1.333
10	$u(y, x_i)$		0.30	-0.0507	0.222
11	$u(y)^2$, $u(y, x_i)^2$	0.1420	0.09	0.00257	0.04938
12					
13	$u(P_{op})$	0.377	(0.377/1.111 = 0.34 as a relative standard uncertainty)		