

## How to interpret an ANOVA table?

# ANOVA

ANOVA involves the calculation and interpretation of a number of parameters which are all summarized in a table. In practice, the calculations are best carried out using statistical software or a MS Excel® spreadsheet. Before going into details of calculations from the first principle, let's take a look at a typical output from a one-way ANOVA in a results table form, whether manual or using software. We will also see how the results are interpreted.

The general form of a results table from a one-way ANOVA, for a total of  $N$  observations in  $k$  groups is shown in Table 1 below.

**Table 1: Results table from *one-way* analysis of variance**

<i>Source of variation</i>	<i>Sum of squares SS</i>	<i>Degrees of freedom, <math>\nu</math></i>	<i>Mean square MS</i>	<i>F</i>
Between-group	$SS_b$	$k - 1$	$MS_b = SS_b / (k - 1)$	$MS_b / MS_w$
Within-group	$SS_w$	$N - k$	$MS_w = SS_w / (N - k)$	
Total	$SS_T = SS_b + SS_w$	$N - 1$		

The above table shows 3 rows relating to different sources of variation and a number of columns containing calculated values related to each source of variance.

Row 1 relates to variation between the means of the groups; the values are almost always either referred to as “between group” terms or are identified by the ‘grouping factor’. For example, data from different analysts are grouped by ‘analyst’. We can also be more specific and label it as an “analyst effect” or “between-analyst”.

Row 2 refers to variation *within* each group (or analyst, for example), meaning the sample standard deviation of each group as compared with the overall performance of other group members. Several different terms may be used to describe this within group variation, “within-group”, “residual”, “error”, or “measurement” being the most popular.

Row 3 on “Total” is not always given by software, but it is fairly consistently labelled “Total” when present.

Under the column section of the ANOVA table, we have the following subjects:

### **Sum of squares (SS)**

The SS terms are calculated by adding a series of squared error terms.

In the case of within-group SS term  $SS_w$ , we are interested in the differences between the individual data points and the mean of the group to which they belong.

For the Total SS term, it is the difference between the individual data points and the mean of all the data (the “grand or overall mean”) that is of interest.

The between-group SS term is conveniently the difference between the Total SS and the within-group SS.

### **Degrees of freedom $\nu$**

For the one-way ANOVA in the table 1, the total number of data points is  $N$  and the number of groups of data is  $k$ . The total number of degrees of freedom is  $N - 1$ , just as for a simple data set of size  $N$ .

There are  $k$  different groups and therefore,  $k - 1$  degrees of freedom for the between-group effect.

The degrees of freedom associated with the within-group SS term is the difference between the two values,  $N - k$ .

Of course, if each group of data contains the same number of replicates,  $n$ , then the degrees of freedom for the within-group SS can be simplified to be equal to  $k(n - 1)$  and the total number of observations is  $N - kn$ .

### **Mean squares MS**

The mean squares are the key term in classical ANOVA. They are variances, calculated by dividing the between- and within-group sum of squares by the appropriate number of degrees of freedom. In Table 1,  $M_b$  represents the between-group MS term (sometimes denoted  $M_1$ ) and  $M_w$  represents the within-group MS term (sometimes denoted  $M_0$ ).

The mean squares are the values used in the subsequent test for significant difference between the group means.

These mean squares also allow estimation of the variance components, i.e. the separate variances for each different effect that contributes to the overall dispersion of the data. We shall discuss how this is being done in the coming blog.

### ***F* (or *F* Ratio)**

The mean squares are compared using an *F*-test as indicated in the above table. The hypothesis for the *F* -test in Table 1 are:

$$H_0 : MS_b = MS_w$$

$$H_1 : MS_b > MS_w$$

If all means are 'equal' (or not significantly different), the two mean squares should also be not significantly different and hence  $H_0$  is true.

We expect  $MS_b$  to be equal or greater than  $MS_w$  as it has included an extra element of variance between group and variance cannot be negative. Therefore this *F*-test is a one-tailed test for whether  $MS_b$  is greater than  $MS_w$  with formula  $MS_b/MS_w$ . This is the value shown in the column *F* in Table 1.

No value for *F* is given for the residual mean square, as there is no other effect with which it can usefully be compared.

### ***F*<sub>crit</sub> and *p*-Value**

Many ANOVA tables include one or two further columns, containing the critical value  $F_{crit}$  against which the calculated value *F* is to be compared for a chosen significance level, and a *p*-value including the significance of the test. These are important information for interpreting the outcome. For example if a chosen significance level is set at 0.05, then a calculated *F* value which is less than  $F_{crit}$  at this significance level or a calculated *p*-value which is more than 0.05 would indicate that the  $MS_b$  is not significantly different from  $MS_w$  and hence, the mean values given by the group are not significantly different from each other.