How to apply the LSD method?

In the last blog on "Study of one-way ANOVA with a fixed-effect factor", we asked when we encountered a significant difference amongst the samples analyzed within a factor, what the reasons for the observed difference were. Was it one mean differed from all the others? Could all the means differ from each other? Might there be a case that the means fell into two distinct groups?

A simple way of deciding the reasons for this observation is to arrange the means in increasing order and compare the difference between adjacent values with a quantity called the **least significance difference** (LSD) which is:

$$LSD = s\sqrt{\frac{2}{n}} \times t_{h(n-1)}$$

where s is the within-sample estimate of σ_o^2 and $h(n-1)$ is the number of degrees of freedom of this estimate.

Let's reproduce the means of all 6 analysts in the example given in the last blog:

Table 1: The means of results reported by the 6 analysts

Analyst	A	B	C	D	E	F
Mean	84.556	84.206	84.410	84.226	84.178	84.390

The rearranged means in ascending order are as follows:

Table 2: Rearranged results in ascending order

Analyst	E	B	D	F	C	A
Mean	84.178	84.206	84.226	84.390	84.410	84.556

Adjacent differences calculated from the analysts' means are shown in Table 3 below:

Table 3: Calculated mean differences amongst the analysts

Analyst	Mean	From A	From B	From C	From D	From E	From F
A	84.56	0	-0.350	-0.146	-0.330	-0.378	-0.166
B	84.21	0.350	0	0.204	0.020	-0.028	0.184
C	84.41	0.146	-0.204	0	-0.184	-0.232	-0.020
D	84.23	0.330	-0.020	0.184	0	-0.048	0.164
E	84.18	0.378	0.028	0.232	0.048	0	0.212
F	84.39	0.166	-0.184	0.020	-0.164	-0.212	0

Recall the within-sample estimate of σ_o^2 by averaging these six analysts' variance values gave us a value of 0.04416. The $s = \sqrt{(0.04416)} = 0.2041$.

Hence, the LSD = $0.2041 \times \sqrt{(2/5)} \times t_{6(5-1)} = 0.2041 \times \sqrt{(2/5)} \times 2.06 = 0.266$

Comparing this value with the differences between the means in Table 3 reveals that the analyst A gave a mean result which differ significantly from analyst B, D and E instead of all the analysts. The other analysts seemed to have generated comparable mean results.

It may be noted that the F value in the ANOVA carried out on this example was 2.649 which was only slightly greater than its F critical value of 2.621 with a p-value of 0.048 ~ 0.05. This marginal significance test value found might have accounted for the observation that not all the other analysts differed their means significantly against that of analyst A.

This least significant difference method is not entirely rigorous as it might lead to rather too many significant differences amongst the samples in the factor. However, it serves as a simple follow-up test when ANOVA has indicated that there is a significant difference between the means. There are other more rigorous tests for consideration such as Bonferroni's, Sidak's, Dunnett's, etc.