The arithmetic of ANOVA calculations

In the recent blog on "Study of one-way ANOVA with a fixed-effect factor", we went through the calculation of one-way or single-factor ANOVA in details in order to make the principles behind the method clearer. However, when you get more well verse of the ANOVA application, it is definitely a good practice to present the ANOVA results in a more organized format instead of a narrative presentation like the last blog did for the sake of easy reading and appreciating the logics and conclusions.

Using the same symbols as given in the last blog, we can first summarize the relationship between the sources of variation as illustrated in Table 1:

Table 1: Summary of sums of squares and degrees of freedom

Source of variation	Sum of squares	Degrees of freedom
Between-sample	$n\sum_{i-1}^{h}(\bar{x}_i - \bar{\bar{x}})^2$	h - 1
Within-sample	$\sum_{i-1}^{h}\sum_{j=1}^{n}(x_{i,j} - \bar{x}_i)^2$	$h(n-1)$
Total	$\sum_{i-1}^{h}\sum_{j=1}^{n}(x_{i,j} - \bar{\bar{x}})^2$	hn - 1

In fact it is quite easy to calculate the total estimate of σ_o^2 which depends on the within- and between- sample variation, as it is a sum of all the squares of the deviation of values from the overall mean $\bar{\bar{x}}$ and divided by the number of degrees of freedom (hxn - 1).

Having done with the calculations for within- and between-sample estimates, we can then proceed to do the F-statistic test, check the corresponding F-critical value and calculate the p-value like most computer software such as Minitab®, SPSS®, Excel®, etc do.