What are Mandel's k / h test statistics?

Objectives of Mandel's *k/h* consistency test statistics

- *k* and *h* test statistics are measures for data consistency, particularly useful for inter-laboratory studies
- By studying the collated data deviations and accuracy, the performance of a laboratory in terms of its reliability and errors can be established
- The laboratory with poor performance can then do its own in-house investigation and make corrective actions for such deficiencies

Mandel's k and h consistency test statistics are discussed in ASTM E691 standards for interlaboratory analysis :

"*Standard practice for conducting an interlaboratory study to determine the precision of a test method*"

Inter-laboratory cross-checks and proficiency testing programs

- Inter-laboratory comparison of test results is an efficiency way to validate the precision of a test method and also to compare the technical competence of the laboratory personnel in terms of precision and accuracy
- Many participating laboratories will carry out series of analyses on one or more given similar samples at about the same period. The data collated are statistically analyzed

Evaluating k test statistic

- k value is a measure of *within*–laboratory consistency in repeatability

- If there are p number of participating laboratories (j), and n is the number of repeats in a laboratory $(x_1, x_2, x_3, \ldots x_i \ldots, x_{n-1}, x_n)$

- The k value of lab (j) is :

$$k_j = \frac{s_j}{s_r}$$

$$s_j = \sqrt{\frac{\sum_{i=1}^{n}(x_{i,j} - \overline{x}_j)^2}{n-1}}$$

$$s_r = \sqrt{\frac{\sum_{j=1}^{p} s_j^{\ 2}}{p}}$$

Interpretation of *k* value

- The *k* value compares the repeatability standard deviation of a laboratory data set with the average of the repeatability standard deviations of all other laboratories
- From the *k* value, we can evaluate the spread of the data set and its precision。
- This test statistic reflects the *single* lab’s repeatability against the average repeatability of all participating laboratories
- The larger the *k* value, the bigger is the data deviation, indicating the poorer the precision

k critical value for consistency (*k-crit*)

- *k-crit* value is the critical value of seriousness for data deviation at a given probability
- *k-crit* defines as:

$$k-crit = \sqrt{\frac{p}{1+(p-1)/F}}$$

- where: F value is from the $F-F$ distribution,

  p is the number of participating laboratories。

- When the k value is higher than the *k-crit*, it can be concluded that the test result deviation is serious with poor precision and unacceptable.

How to obtain the *F* test statistic value?

- $F\ (v1,\ v2)$ is the $F-F$ distribution value
- Degree of freedom $v1 = (n\text{-}1)$, n is the number of repeats in a single laboratory
- Degree of freedom $v2 = (p\text{-}1)(n\text{-}1)$
- Upon knowing the degrees of freedom, we can obtain the *F* value from the F–F table
- Or use the Excel function "=FINV(0.05,*v1*,*v2*)"

Evaluating the *h* test statistic

- The *h* test statistic is used to examine the consistency of inter-laboratory data, confirming if any laboratory data is an outlier
- In other words, it is to indicate the accuracy of a lab results against the others reported
- Let p be the number of participating labs with the lab mean results as follows: $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_j, \ldots \overline{x}_p)$
- The overall mean result of this interlaboratory study is : $\overline{\overline{x}} = \frac{\sum_{j=1}^{p} \overline{x}_j}{p}$

Evaluating the *h* test statistic

- The deviation of mean result of a lab (j) from the overall mean is:

$$d_j = \overline{x}_j - \overline{\overline{x}}$$

- The standard deviation of these comparison is : $$s_{\overline{\overline{x}}} = \sqrt{\frac{\sum_{j=1}^{p} d_j^{\,2}}{p-1}}$$

- The h value of lab (j) is : $$h_j = \frac{d_j}{s_{\overline{\overline{x}}}}$$

Interpretation of h test value

- h test statistic value reflects the deviation of a single laboratory's mean test results from the overall mean results obtained from all participating laboratories
- The larger the h value, the bigger the deviation, the poorer is the accuracy of that single laboratory

h critical value for consistency (*h-crit*)

- *h-crit* is a measure of seriousness in a lab's inaccuracy
- *h-crit* defines as:

$$h - crit = \pm \frac{t(p-1)}{\sqrt{p(t^2 + p - 2)}}$$

- where: t is the Student's distribution with degree of freedom $v = p - 2$, and $\alpha = 0.05$;

  p is the number of participating laboratories

When the h value is larger than the *h-crit*, it is concluded that the mean result given by the laboratory concerned is not accurate and reliable

Mandel's曼德尔 k / h 统计测验量

原始数据

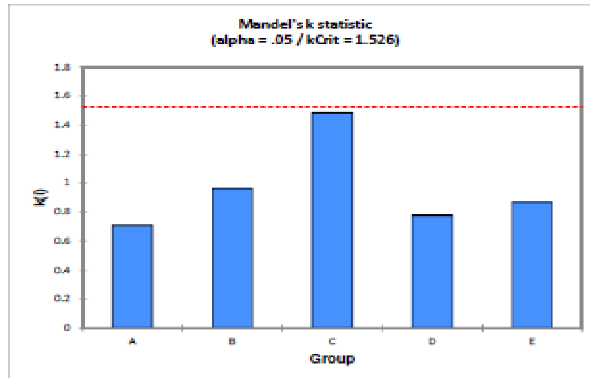
实验室：	A	B	C	D	E
	9.35	9.06	8.86	9.84	8.88
	10.12	9.86	8.34	9.56	9.98
	9.32	10.20	10.33	9.43	8.86
	9.14	10.40	9.86	10.50	9.45

k 测验

实验室：	A	B	C	D	E
标准差 s(j)	0.435	0.590	0.909	0.477	0.534
方差 s^2	0.189	0.349	0.827	0.227	0.285
总方差	1.877				
实验室数目，p	5				
均总方差	0.375				
均总标准差	0.613				
k - 值	0.710	0.964	1.484	0.778	0.871

F-值 [v1=(4-1); v2=(5-1)(4-1)]	3.490
k-crit 临界值	1.526

$$k\text{-}crit = p / [\ 1 + (\ p\ - 1) / F\ ]$$



h 测验

实验室：	A	B	C	D	E
平均值	9.483	9.880	9.348	9.833	9.293
总平均值	9.567				
均值差，d	-0.085	0.313	-0.220	0.265	-0.275
实验室数目	5				
方成均值差 d^2	0.007	0.098	0.048	0.070	0.075
总方成均值差	0.299				
总均值标准差	0.273				
h-值	-0.309	1.145	-0.803	0.971	-1.004

t 值 （v=5-2 = 3)	3.182
+/- h-crit 临界值	1.571

$$h - crit = \frac{t(p-1)}{\sqrt{p(t^2 + p - 2)}}$$