How to use R in paired t-testing?

In a testing laboratory, we can encountered several situations in which we have paired data collected. For example, we wish to compare our newly developed test method against a reference or standard method. We may also want to compare the testing skill of a new recruit against that of our skilled supervisor through repeated analysis of a sample.

In these cases, the basic model is $Y_i = X_i + \varepsilon_i$ where ε_i is its random error. We are interested in knowing if mean ε_i is zero. Hence, we will look at the error (difference) $d_i = Y_i - X_i$ and use the Student's *t*-test for significance testing.

Here we have two means of a series of testing, say μ_1 and μ_2.

Our significance hypothesis are therefore:

$$H_o : \mu_1 = \mu_2 \; ; \quad H_1 : \mu_1 \neq \mu_2$$

Under the assumption of a normal distribution, $d_i = Y_i - X_i$ which approximates $N(\mu, \sigma_d^{\,2})$ where $\mu = \mu_1 - \mu_2$ and $\sigma_d^{\,2} = \sigma_1^{\,2} + \sigma_2^{\,2}$. For these paired values, we will test statistically if μ is zero, i.e. we write:

$H_o : \mu = 0$; $H_1 : \mu \neq 0$ with a given α significance level.

Use the following equation:

$$t = \frac{\bar{d}}{(s_d / \sqrt{n})}$$

where, $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$; $d_i = y_i - x_i$; $s_d = \sqrt{\left[\frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2\right]}$

Calculate the *t*-value and its probability p-value. If the t-value falls within the rejection or critical region at $\sim t(n-1)$, we shall reject H_o and assume there is a significance difference between the mean results of these two paired results.

For example, in a new method development exercise, we have a series of test results obtained by the new method and another series of data from a reference method, as shown below:

New method	23.5	25.6	24.9.	25.1	24.6
Reference method	24.8	25.1	25.5	24.3	24.6

Before carrying out the t-test, we first test if the two sets of values have comparable variance by using the *F*-test.

The R programming gives the following results:

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> New=c(23.5,25.6,24.9,25.1,24.6)
> Reference=c(24.8,25.1,25.5,24.3,24.6)
> var.test(New,Reference)

        F test to compare two variances

data:  New and Reference
F = 2.8779, num df = 4, denom df = 4, p-value = 0.3304
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
  0.2996434 27.6412057
sample estimates:
ratio of variances
          2.877934

> t.test(New,Reference,paired=TRUE)

        Paired t-test

data:  New and Reference
t = -0.31689, df = 4, p-value = 0.7672
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.1713896  0.9313896
sample estimates:
mean of the differences
                  -0.12
```

The above results showed that the variance of the two methods were comparable with p-value = 0.3304>0.05, and its *t*-value calculated had a p-value of 0.767 which is larger than 0.05, indicating the Ho cannot be rejected, i.e. indicating that the two mean results were statistically similar.