Understanding the Binomial Probability Distribution (Part II)

Let's now take one step further to talk about expected value and variance of the binomial probability distribution.

The expected value *E*, or mean μ, of a random variable is a measure of the central location for the random variables in hand. The general formula for the expected value of a discrete random variable x is:

$$E(x) = \mu = \Sigma x f(x)$$

That means in order to obtain the expected value, we have to multiply each x value of the random variables by the corresponding probability $f(x)$ and then add the resulting products. Hence simply for *a* discrete random variable x, we write

$$E(x) = \mu = np$$

where n = number of trials and p = a known probability of success

For example, when we flip a fair coin 8 times, the probability of getting a Head is p=0.5. Hence, we would expect the mean number of heads to be half of the flips, or np = 8 x 0.5 = 4.

We use variance as a tool to measure the variability, or dispersion of the random variables obtained. The general variance formula of a discrete random variable is :

$$\mathrm{Var}(x) = \sigma^2 = \Sigma(x-\mu)^2 f(x)$$

For a discrete random variable x, we have:

$$\mathrm{Var}(x) = \sigma^2 = np(1 - p)$$

If we roll a perfect die for 20 times with a p = 1/6 = 0.1667, our variance Var(x) is therefore:

$$\mathrm{Var}(x) = 20 \times 0.1667 \times (1–0.1667) = 2.78$$

and the standard deviation $\sigma = \sqrt{(2.78)}$ = 1.67. Of course, the mean of success = np = 20 x 0.1667 = 3.33

Now, when the number of trials getting bigger, evaluating the discrete binomial probability function by hand or even with a calculator is becoming cumbersome. Hence, when we encounter a binomial distribution problem with a large number of trials, we would like as usual to approximate it because it is approaching a continuous normal probability distribution.

This result follows from the Central Limit Theorem. The mean and variance for the approximately normal distribution of x are np and $np(1\text{-}p)$, identical to the mean and variance of binomial(n,p) distribution.

In fact, a rule of thumb to use the approximate normal distribution when the number of trials n is greater than 20, $np \geq 5$ and $n(1\text{-}p) \geq 5$. Therefore, the normal distribution provides an easy-to-use approximation of binomial probabilities in order to save time and labor in calculation..

What we can do is to set $\mu = np$ and $\sigma = \sqrt{np(1-p}$ in the definition of the normal curve. Let's see again our familiar experiment of rolling a perfect die with n = 50 and p = 1/6 = 0.1667.

In applying the normal approximation to the binomial, we set $\mu = np =$ 50(0.1667) = 8.33 and $\sigma = \sqrt{np(1-p}$ = 2.64.

In fact, to use the normal approximation to calculate this binomial probability, we should first acknowledge that the normal distribution is *continuous* and apply the ***continuity correction***. This means that the probability for a single discrete value, say 100, is extended to the probability of the *interval* (between 99.5 and 100.5). Because we are interested in the probability that x is less than or equal to 100, the normal approximation applies to the upper limit of the interval, 100.5. If we were interested in the probability that x is strictly less than 100, then we would apply the normal approximation to the lower end of the interval, 99.5.