An alternative approach for GUM in the estimation of measurement uncertainty – Monte Carlo simulation

In addition to the popular GUM approach for evaluating measurement uncertainty in testing, there are growing interests in the holistic top-down approach to estimate measurement uncertainty based on the overall method performance, i.e. looking at the method precision, accuracy and trueness. The main reason for such a change is that GUM method has been found to be tedious in the estimation process because testing procedures have more steps to take care and each step might have certain uncertainty to be considered.

We know that the GUM uncertainty framework essentially uses the law of propagation of uncertainty to propagate expectations and standard uncertainties associated with the uncertainty contributing factors as inputs through to an output quantity (measurand). Recently, the Monte Carlo method using the propagation of distribution has also been considered for estimating measurement uncertainty. A good document of JCGM 101:2008 can be referred to, as a supplement to GUM.

But what is a Monte Carlo method?

Basically, Monte Carlo method is a technique which involves using random numbers as inputs and probability to solve problems. Before we discuss the Monte Carlo simulation, let's understand what a computer simulation is about.

Computer simulation uses computer models to imitate real life or make predictions of an event. If we have a model with some mathematical equations, we can use a spreadsheet such as Excel to insert a certain number of input parameters to give a set of outputs or response variables. Such model is usually deterministic, meaning that we get the same results no matter how many times we re-calculate.

Monte Carlo simulation (MCS) on the other hand is a method for iteratively evaluating a deterministic model using a very large population of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than few uncertainty parameters. A simulation using a computer can typically involve over 10,000 evaluations or more of the model.

By doing so, MCS can be a good way to establish the validity of a statistical formula at its basic. This can usefully complement the theoretical approach of traditional statistics. In particular, an expression for the estimate of the population variance can be established from theoretical considerations. MCS can be effective in comparing actual and theoretical values for the estimate of the population variance.

Readers who are familiar with the Eurachem Guide on measurement uncertainty for testing can have a look at a simple example quoted in the Guide, dealing with the preparation of a calibration standard. It presents the MC technique with detailed step-by-step manner. In this case, the results obtained by both GUM and MC approaches are quite similar.

In summary, Monte Carlo method uses computer to generate huge random numbers of stated distribution which estimate the mean values of the input quantities for the calculation of its output quantity.