

What is a P -value in a test statistic?

Have you noticed that the results of ANOVA, F - and t -tests of the Data Analysis pack provided by the Microsoft Excel spreadsheet have come with a $P(F \leq f)$ value? What is the significance of this P -value and how is it estimated?

In fact most statistics software such as SPSS, Excel, etc. describe the acceptance of a test statistic result with a P -value obtained at a certain confidence interval. So, what does this P -value mean and what is its role in deciding if a null hypothesis H_0 of a significance testing is to be rejected or not?

The answer is that if the null hypothesis were to be true (i.e. no effect on the treatment or factor), the P -value shows the probability of obtaining data as extreme as was observed. In other words, if the P -value is smaller than our chosen significance level (α), we shall reject the null hypothesis. For example, if we have given a significance level of 0.05, any P -value of <0.05 indicates that the null hypothesis is NOT true, which means the treatment does have an effect on the final outcome.

For example, each of the two laboratory analysts carried out a series of cyanide analysis in a given waste water sample. Due to the personal skill difference, we would like to test if the data precision of the more experienced Analyst B in terms of variance was better than that of Analyst A by carrying out a F -test. The raw data in mg/L obtained are summarized as below:

#	Analyst A	Analyst B
1	8.65	8.16
2	8.12	8.32
3	9.28	7.92
4	8.85	8.66
5	8.94	8.86
6	8.33	8.04

In here, the \bar{x} (Analyst A) = 8.695 mg/L with sample standard deviation s_A of 0.422 mg/L; \bar{x} (Analyst B) = 8.327 mg/L with a standard deviation s_B of 0.366 mg/L.

For the one-tailed hypothesis testing, we state at a significance level $\alpha = 0.05$,

$$H_0: s_A = s_B$$

$$H_1: s_A > s_B$$

The MS Excel F -test (two samples for variance) gives the following results:

F -Test Two-Sample for Variances

	Analyst A	Analyst B
Mean	8.695	8.327
Variance	0.178	0.134
Observations	6	6
Df	5	5
F	1.330	
$P(F \leq f)$ one-tail	0.381	
F Critical one-tail	5.050	

As the estimated F -value which was the ratio of variances $0.178/0.134$ was found to be 1.330 which was smaller than the critical F value of 5.050 at $\alpha = 0.05$, $Df_A = 5$, $Df_B = 5$, the H_0 hypothesis was accepted, i.e. there was no significance difference between the two variances. The P -value accordingly was also noted to be 0.381 which was of course larger than $\alpha=0.05$.

This P -value was calculated by the Excel function: “=F.DIST.RT(1.330,5,5)” .