DOE – Concept of interaction in factorial designs

Most experimental designs do not confine to only one factor treatment but usually have to consider more than one factor simultaneously. These experimental designs have been classified under the name of factorial designs, because they evaluate the effects of two or more factors at the same time.

The simplest factorial design isthe two-factor design with two levels for each factor of interest. Designs which contain more than two levels of a factor are logical extensions of the two-level case. We can consider the case where there are equal numbers of replicates (n) for each combination of the levels of factor *A* with those of factor *B*.

It is clear that as the number of levels of each factor increases and the number of replications in each factor also increases, we will encounter quite complex calculations. Hence, it is assume that in practice, a statistical software such as SPSS, MINITAB or Excel spreadsheet package of ANOVA will be used when analyzing data from such experimental design models.

Upon the analysis of variance, we will see the significance of factor A, factor B, and their interaction, if any. But, what exactly do we mean by the term "interaction"? How do we know if there is any interaction between these two factors? To answer these questions, we have to understand the interpretation of the concept of interaction.

We can simply put it in a simple perspective:

"If there is no interaction between two factors (A and B), then any difference in the dependent or response variable between the two levels of factor A would be the same at each level of factor B."

For example, in a hypothetical experiment to study the yields of a synthesized chemical in gm on two types of catalysts (X and Y) and at two reaction temperatures (40°C and 80°C), our factors are catalysts and temperatures of reaction. We may get one of the four scenarios as depicted in the following graphs by plotting the average yield values for each catalyst for each temperature of reaction:

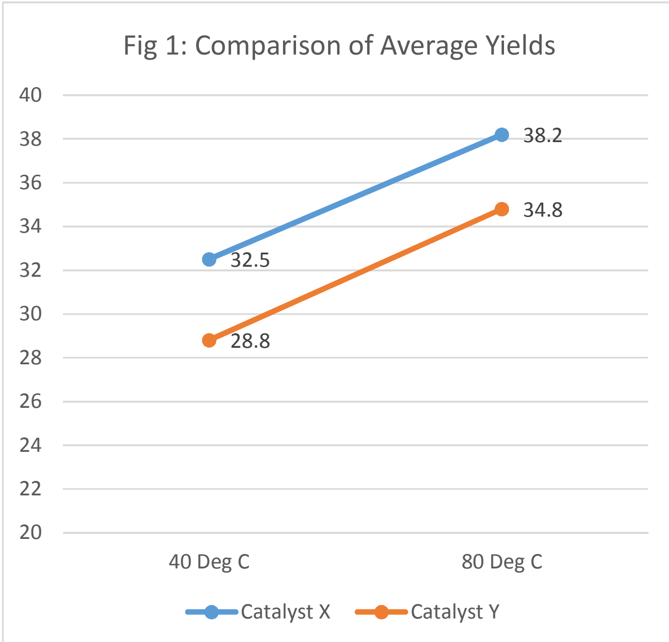
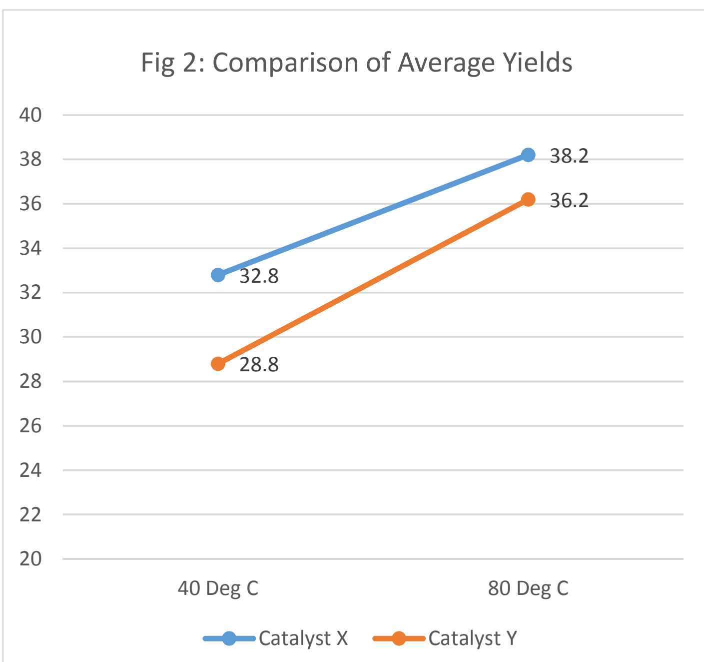
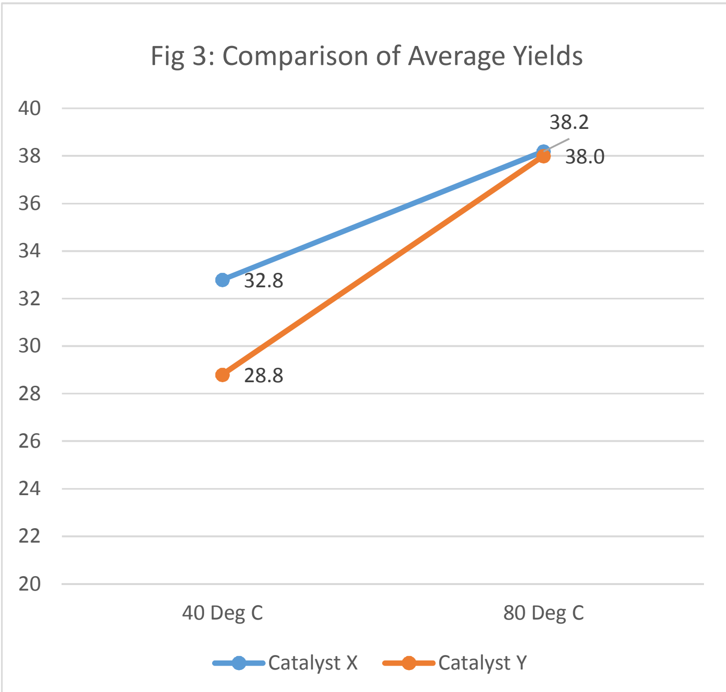


Figure 1 shows almost parallel lines of chemical yields at two different temperatures for these two catalysts X and Y. This indicates that the test for the interaction to be non-significant, and therefore the differences observed between the two catalysts at each reaction temperature is purely a sample effect or due to chance.



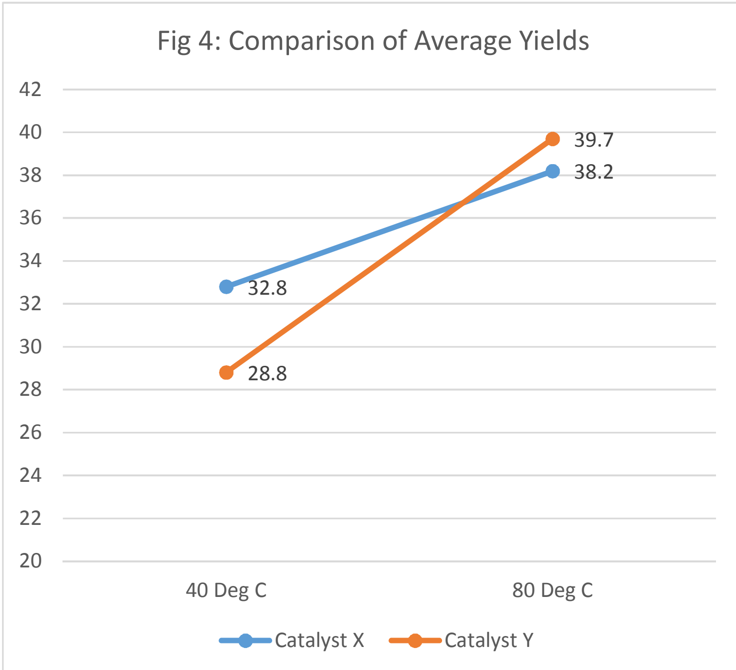
In terms of the factors in this example, if there were no interaction between the catalysts and reaction temperatures, any difference between catalyst X and catalyst Y would be the same under conditions of 80°C as it is under conditions of 40°C. In Fig 2, it is observed that the average difference between catalysts when at 80°C (2.0gm) is 4.0gm less than the average difference between catalysts for 40°C (6.0gm). The ANOVA would have found the test for the interaction to be insignificant, as also reflected in the Figure 2.

We can contrast the parallel lines of Figure 1 with a different case in which there is a strong interaction as illustrated in Figure 3 below:



The results depicted in Figure 3 represent a pronounced interaction between the two factors, taking the possible variations in mind. It is clear that any difference between the catalysts is different in two reaction temperatures.

We can discuss the concept of interaction further by supposing that the average amount of yield by catalyst Y at 80°C was 39.7mg, instead. Figure 4 below shows the interaction graph for this situation where we observe a crossing pattern in the lines for the two catalysts. It is obvious to conclude that catalyst Y is a better catalyst at 80°C in terms of yield value whilst catalyst X produces more yield at lower reaction temperature.



However, it must be cautioned that whilst interaction plots are extremely helpful in interpreting the analysis of an experiment, they give no indication of the size of the experimental error as seen in the Figure3 at 80°C reaction temperature. Sometimes a perceived interaction in the plot will not be distinguishable from error variability in the analysis of variance. On the other hand, if the error variability is very small, then an interaction effect may be statistically significant in the analysis, even if it appears negligible in the plot.