Decision risks in conformance testing – Part IV

Other approaches for decision rules

Apart from setting a targeted risk probability value P (such as 5% and 2%) for making false acceptance or false rejection, respectively in compliance testing based on comparing the test tolerance and the accuracy of the measuring instrument, we can also employ other simpler approaches for decision rules but with certain conditions.

a. Measurement capability index (Cm)

When the MPE or tolerance limits are symmetrical (i.e. MPE+ = MPE−), whilst risk of false acceptance (P_{fa}), measured E_I and calculated measurement uncertainty u_{EI} are all known, we can make a relatively "quick" assessment on a test on a measuring instrument or system by using the concept of measurement capability index, which is defined as:

$$Cm = \frac{MPE}{2 \times u_{EI}} \quad \text{or} \quad Cm = \frac{MPE}{U_{EI(k=2)}} \qquad (1)$$

Note: u_{EI} is the standard uncertainty of the error of indication, and U_{EI} is the expanded uncertainty with a coverage factor of 2 for 95% confidence.

Mathematically, the equation (1) is derived from the original Cm, defined for MPE limits MPE- and MPE+ as:

$$Cm = \frac{(MPE-) + (MPE+)}{2(2 \times u_{EI})} = \frac{MPE}{2 \times u_{EI}} \qquad (2)$$

This Cm is also known as the Test Uncertainty Ratio (**TUR**) and is relevant for all measurement applications and calibration arena. By accounting rigorously for all relevant uncertainty components, and with an adequate ratio value, the Cm or TUR can provide for effective measurement quality assurance for most measurement scenarios.

A risk-based decision rule has to establish a specific confidence level and also acceptance limits for conformance testing.

Scott Mimbs of NASA presented Table 1 below for three conformance (in-tolerance) confidence levels, in his paper "*Conformance testing: Measurement decision rules*" at the 2010 NCSL International Workshop and Symposium. This approach allows for difference confidence levels depending on both how critical is for the measurand and the capability of the measurement process.

Table 1: Percentage of usable conformance for a desired confidence level

TUR	Conformance confidence level	Percentage of conformance
10 : 1	99.7%(+3σ)	86.3%
4 : 1	99.7%(+3σ)	65.7%
1: 1	99.7%(+3σ)	Not possible
10 : 1	95.45%(+2σ)	91.5%
4 : 1	95.45%(+2σ)	78.9%
1: 1	95.45%(+2σ)	Nominal only
10 : 1	68.3%(+1σ)	97.6%
4 : 1	68.3%(+1σ)	94.0%
1: 1	68.3%(+1σ)	76.2%

The NASA Handbook NASA-HDBK-8739.19-4:2010 "*Estimation and evaluation of measurement decision risk*" states that "Where it is not practical to compute false acceptance risk, the (ANSI/NCSL Z540.3-2006) standard requires that the measurement's TUR shall be greater than or equal to 4:1." In this case, the percentage of conformity is 78.9% as shown in Table 1.

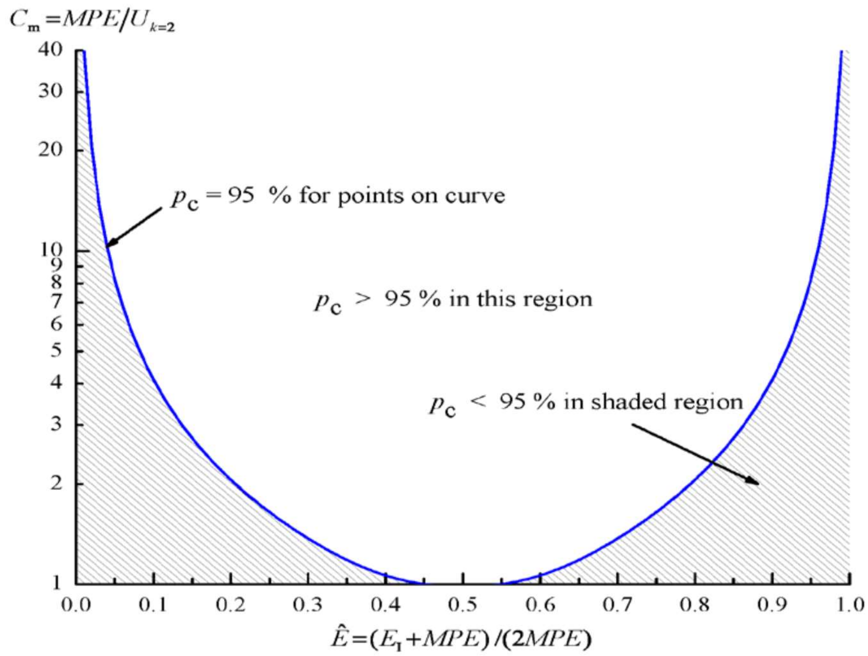
Furthermore, we may apply the Cm in another approach but it is first necessary to calculate another parameter, $\hat{E}$, which is defined as:

$$\hat{E} = \frac{E_I + MPE}{2MPE} \qquad (3)$$

Note: Since E_I is between $\pm$MPE, we expect that $0 < \hat{E} < 1$.

The OIML Guide G19:2017 "*The role of measurement uncertainty in conformity assessment decisions in legal metrology*" reproduces a chart (Figure 1) by W. Tyler Estler, showing the relationship between $\hat{E}$ and *Cm,* constructed for a given conformance probability P_c =95%.

Figure 1: A chart of $\hat{E}$ and measurement capability index *Cm*



From the intersection of $\hat{E}$ and *Cm* , we can find out if the $\hat{E}$ value lies in the shaded region which is less than 95% confidence level to show the test fails, or unshaded region with more than 95% confidence to indicate the test passes.

b. Guard bands

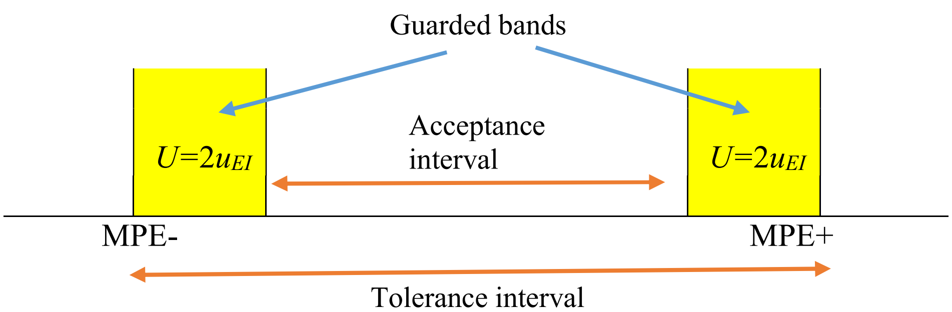
When risk of false acceptance or false rejection is used where the uncertainty of the error of indication (u_{EI}) is deemed constant, a particularly convenient method can be used for making conformity decisions, known as "guard banding".

Under such conditions, we can create an *acceptance limit* (also known as a guard band) by simply "shift" the MPE boundaries *inward* (for false acceptance) or *outward* (for false rejection) by an amount equal to the respective risks, and conformity decisions are then made on the basis of whether the measured value of error of indication lies within or outside of the shifted conformity boundaries or acceptance zone.

Hence, by setting an acceptance limit or guard band *inside* the MPE boundaries, we reduce the risk of accepting a non-conforming instrument or system. Such tolerance zone created is also known as *stringent acceptance zone*. It is positive compliance for acceptance.

See Figure 2 for an illustration of the guarded acceptance.

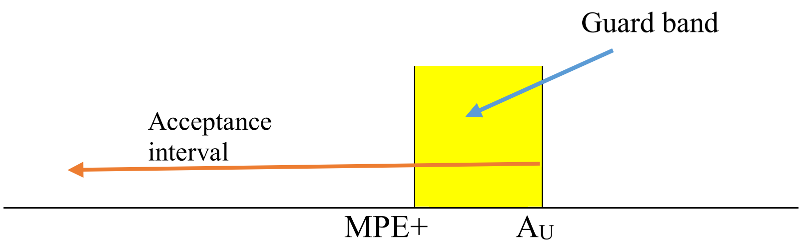
Figure 2: Two sided acceptance interval created by reducing the MPE boundaries on either side by the k=2 expanded uncertainty $U=2u$



Conversely, if an acceptance limit is chosen *outside* the MPE region, as shown in Figure 3, we can increase the risk probability that a rejected measurand is truly non-conforming. The guarded rejection is also known as *relaxed rejection* and positive non-compliance for rejection.

We employ such a guarded rejection decision rule when we want clear evidence that a limit has been exceeded prior to taking any negative or rejection action.

Figure 3: An upper acceptance limit A_U outside the MPE+ with a rejection guard band



Choosing acceptance and rejection zone limits

Eurachem/CITAC Guide on "*Use of uncertainty information in compliance assessment*" (2007) states that "The size of the guard band g depends upon the value of the uncertainty and is chosen to meet the requirements of the decision rule.".

If the decision rule states that for non-compliance, the observed value should be greater than the limit plus $2u$, then the size of guard band is $2u$. In here, the u is the standard uncertainty.

If the decision rule says that for non-conformance that the probability P that the value of the measurand is greater than the limit MPE, should be at least 95%, then the guard band g is chosen so that for an observed value of $\mathrm{MPE}+g$, the probability that the value of the measurand lies above the MPE is 95%.

Similarly, if the decision rule is that there should be at least a 95% probability that the value of the measurand is less than MPE, then g is chosen so that for an observed value of $\mathrm{MPE}-g$, the probability that the value of the measurand lies below the limit is 95%.

In general, the value of g will be a function of or a simple multiple of u. In some cases, the decision rule may specify the value of the multiple to be used. In others, the value of P determines the guard band.

Conclusions

In order to decide whether or not to accept/reject a product, we need first of all to have a specification giving the upper and/or lower permitted limits (such as MPEs for measuring instrument, or pesticide residual limit for a food product) of the characteristics (measurand), and, secondly a decision rule taking the measurement uncertainty into account with regard to accepting or rejecting a product according to its specification and the measurement result.

The decision rule is based on the size of the acceptance or rejection zone, determined by means of appropriate guard bands, calculated from the value of measurement uncertainty and the minimum acceptable level of probability P that the measurand lies within the specification limits.

More importantly, a reference to the decision rules used should be included when reporting on compliance, in order to avoid any ambiguity or misunderstanding.