

## Decision risks in conformance testing – Part III

### Types of risks associated with conformity decisions

There are three fundamental types of risks associated with the uncertainty approach through making conformity decisions for tests which are based on meeting tolerance interval requirements such as MPEs for measuring instruments. Conformity decision rules can then be applied accordingly.

In summary, they are:

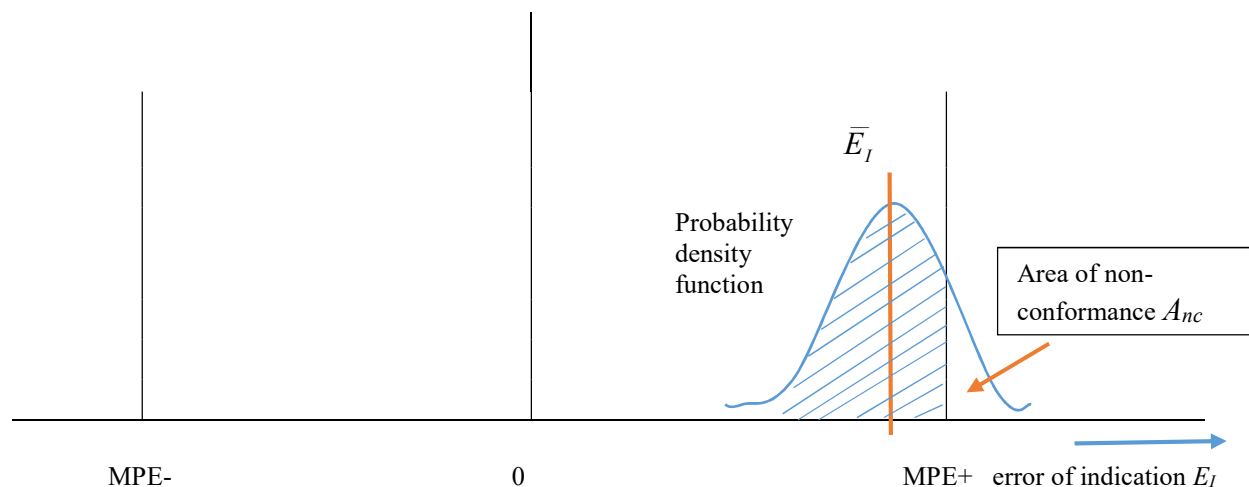
- 1) Risk of false acceptance of a test result
- 2) Risk of false rejection of a test result
- 3) Shared risk

#### 1) Risk and decision rule for false acceptance

Risk of false acceptance means that the test is considered to pass but in reality the MPE requirement might not have been met.

In this case, as shown in the Figure 1 below, the measured value of the error of indication  $\bar{E}_I$  lies within the region bounded by the MPEs but the normal PDF (probability density function) extends into the region outside of the region bounded by the MPEs, meaning that the “true” value of the error of indication is believed to have a chance lying outside of the region bounded by the MPEs.

**Figure 1:** Normal PDF of measured value of  $\bar{E}_I$  in the region of  $\pm$ MPE



This false acceptance risk is taken by the evaluator or user of the measuring instrument or system. Hence, it is also known as the “customer’s risk”, which is that the instrument or system is not performing “within specification” even though the test result reported says it is.

The risk of false acceptance is calculated as the area  $A_{nc}$ , under the PDF curve that is outside of the region bounded by the MPEs, which is the unshaded area under the curve as shown in Figure 1.

A possible *decision rule* that we can give is to allow such probability or risk of false acceptance ( $P_{ac}$ ) to be less than some stated value (example, 5% or  $\alpha = 0.05$ ).

This risk of false acceptance would favor the user of the instrument/system, to the detriment of the manufacturer or vendor of the instrument/system, because the mean error of indication  $\bar{E}_I$  would lie within the region bounded by the MPEs, and worse still, could usually not even lie very close to the relevant MPE boundary if the decision rule is to be met.

## 2) Risk and decision rule for false rejection

On the other hand, risk of false rejection means that the test is considered to fail and the instrument performance is rejected though in reality, the MPE requirement might have been met.

In this case, the measured value of the error of indication lies outside the region bounded by the MPEs, but the PDF extends into the region inside of the region bounded by the MPEs.

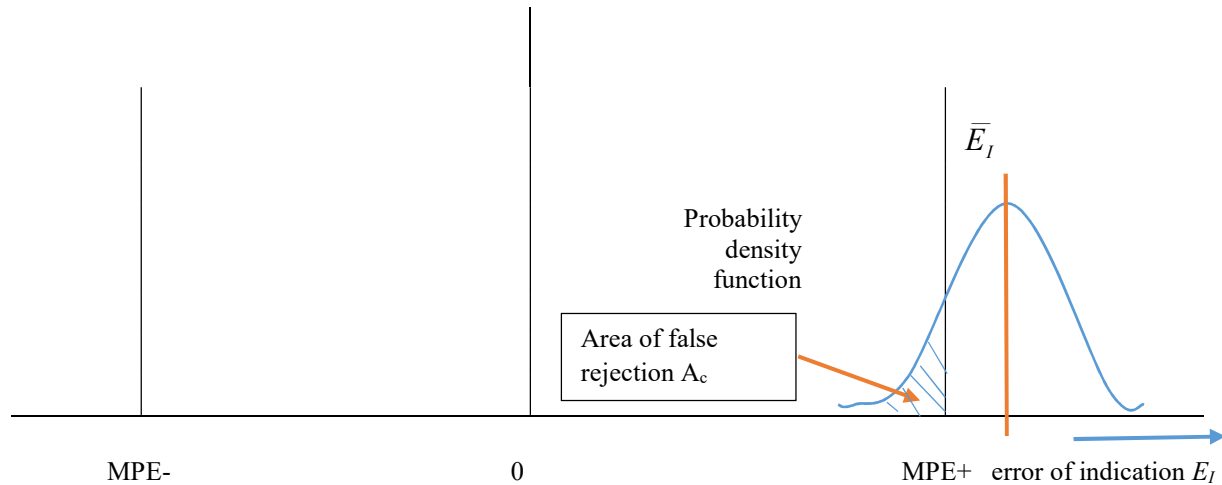
Such risk is taken by the manufacturer or vendor of the measuring instrument or system. So, it is also known as the “producer’s risk”, as the instrument/system is performing “within specification” even though the test result says it is not.

Again, the value of the risk of false rejection is calculated as the area  $A_c$  under the PDF that is inside of the conformance region by the MPEs when the measured value of error of indication lies outside the region bounded by the MPEs.

A possible decision rule made is that the risk of false rejection ( $P_{fr}$ ) be less than some stated value (example, 2% or  $\alpha = 0.02$ ). Such risk would favor the manufacturer or vendor, to the detriment of the customer, since the value of

the error of indication  $\bar{E}_I$  would lie outside of the region bounded by the MPEs, and, further, could usually not even lie very close to the relevant MPE boundary, if the decision rule is to be met.

**Figure 2:** Normal PDF of measured value of  $\bar{E}_I$  outside the region of  $\pm$ MPE



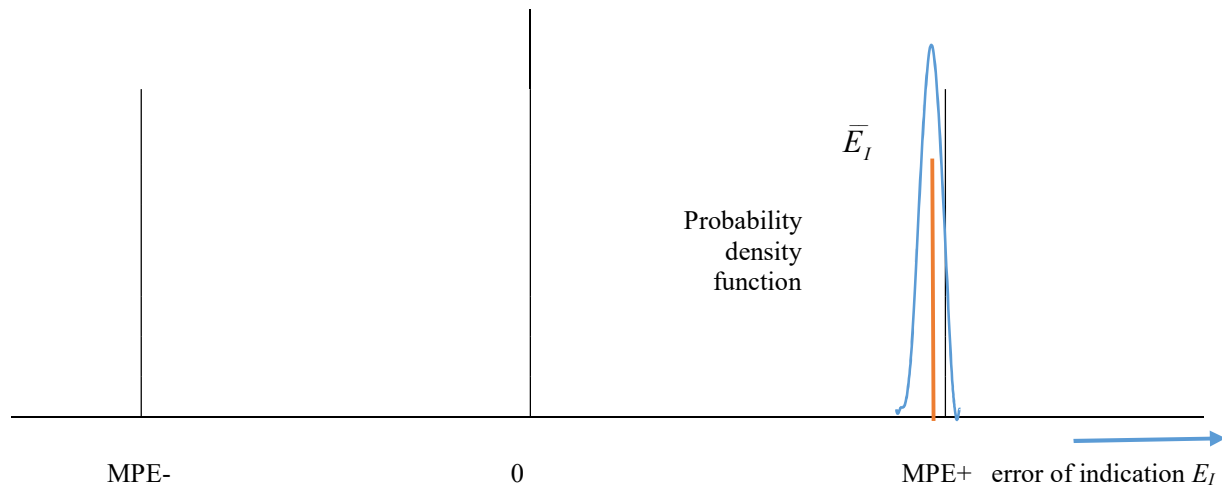
### 3) Shared risk

Shared risk is an agreement between the parties concerned with the outcome of the testing that neither will be given an advantage or disadvantage with respect to the consideration of measurement uncertainty for measured values of the error of indication that are near enough to the MPE boundaries that risk of false acceptance or rejection would be significant.

However it is obvious that an equally shared risk agreement can be reached more readily when the measurement standard uncertainty  $u_{EI}$  is relatively “small” with respect to the MPE (i.e., the ratio  $u_{EI}/MPE$  is small) so that the significant risk of an erroneous decision exists for value of  $\bar{E}_I$  that are only very close to the MPE boundaries.

A large  $u_{EI}$  on the other hand has probably no advantage for a shared risk arrangement. See Figure 3 for an illustration of a small  $u_{EI}/MPE$  ratio for shared risk agreement.

**Figure 3:** Normal PDF of measured value of  $\bar{E}_I$  within the region of  $\pm$ MPE with “small” measurement uncertainty



**How small is small for the ratio  $u_{EI} / \text{MPE}$  for the share risk approach?**

To answer this question, we have to work out what the **maximum permissible uncertainty for the error of indication** ( $\text{MPU}_{EI}$ ) is. This  $\text{MPU}_{EI}$  is thought of as the largest value that  $u_{EI}$  can have for a given measurement of error of indication  $\bar{E}_I$  for which the shared risk approach can be used.

The  $\text{MPU}_{EI}$  is defined as:

$$\text{MPU}_{EI} = f_{EI} \times \text{MPE} \tag{1}$$

where  $f_{EI}$  is a specified number (less than one), usually on the order of 1/3 to 1/5. In other words, the  $\text{MPU}_{EI} : \text{MPE}$  ratio is either 1:3 or 1:5.

Hence, the decision rule is:

“If  $u_{EI}$  is greater than  $\text{MPU}_{EI}$ , then the test is considered to fail, indicating that reducing  $u_{EI}$  (or for incorporating an increased  $\bar{E}_I$  MPE) will need to be developed.”

However, it is equally important to work on the specification of a “**maximum permissible uncertainty ( $\text{MPU}_{std}$ ) for the measurement standard**”, which is the largest value that  $u_S$  is allowed to have for a given measurement of error of indication.

The  $MPU_{Std}$  defined by:

$$MPU_{Std} = f_{Std} \times MPE \quad (2)$$

where  $f_{Std}$  is a specified number (less than one), usually on the order of 1/3 to 1/5. In other words, the  $MPU_{Std} : MPE$  ratio can be between 1:3 and 1:5.

The rationale for this requirement is that if  $MPU_{Std}$  is too large, then pass-fail decision based on  $MPU_{EI}$  above can become dominated by the quality of measurement standard and/or testing laboratory, rather than on the quality of the instrument/system being tested. Remember that  $u_S$  includes  $u_{EI}$  as well as other possible components of uncertainty. This could be considered unfair to test the instrument manufacturer's instrument with a measurement standard that has a large uncertainty, particularly comprising of most of  $u_{EI}$ .

*(In Part IV, we shall see other relatively simpler approaches for decision rules but under specific conditions)*